## PMT

## Mark Scheme

Q1	$E \sim N(406, 12^2)$ When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.					
(i)	$P(E < 420) = P\left(Z < \frac{420 - 406}{12} = 1.1666\right)$	M1 A1	For standardising. Award once, here or elsewhere.	-		
	= 0.8783/4	A1	c.a.o.	3		
(ii)	$C \sim N(406 \times 14.6 = 5927.6,$ $\sigma^2 = 12^2 \times 14.6^2 = 30695.04)$ P(this > 6000) =	B1 B1	Accept equivalent in £. Mean. Variance. Accept sd (= 175.2).			
	$P\left(Z > \frac{6000 - 5927.6}{175.2} = 0.4132\right) = 1 - 0.6602 = 0.3398$	A1	Accept P( <i>E</i> > 6000/14.6) o.e. c.a.o.	3		
(iii)	$B = C_1 + C_2 + C_3 \sim N(17782.8,$ $\sigma^2 = 175.2^2 + 175.2^2 + 175.2^2 = 92085.12)$	B1 B1	Accept equivalent in £, or $E_1 + E_2 + E_3$ . Mean. ft from (ii). Variance. Accept sd (= 303.455). ft from (ii).			
	Require b s.t. $P(B < 100b) = 0.99$ $\therefore \frac{100b - 17782.8}{303.455} = 2.326$	B1	Accept P( $E_1 + E_2 + E_3 < 100b/14.6$ ) o.e. 2.326 seen.			
	303.455 $\therefore 100b = 17782.8 + 2.326 \times 303.455 = 18488.6 (p)$ $b = \pounds 184.89$	A1	c.a.o. (Minimum 4 s.f. required in final answer.)	4		
(iv)	H <sub>0</sub> : $\mu = 432$ H <sub>1</sub> : $\mu < 432$ where $\mu$ is the mean amount of electricity used.	B1 B1	Both hypotheses. Hypotheses in words only must include "population". For adequate verbal definition. Allow absence of "population" if correct notation $\mu$ is used, but do NOT allow " $\overline{X} =$ " or similar unless $\overline{X}$ is clearly and explicitly stated to be a <u>population</u> mean.			
	$\overline{x} = 422.16$ $s_{n-1} = 13.075(4)$	B1	$s_n = 11.936$ but do <u>NOT</u> allow this here or in construction of test statistic, but FT from there.			
	Test statistic is $\frac{422.16 - 432}{\frac{13.075}{\sqrt{6}}}$	M1	Allow c's $\overline{x}$ and/or $s_{n-1}$ . Allow alternative: 432 + (c's -2.015) × 13.075/ $\sqrt{6}$ (= 421.24) for subsequent comparison with $\overline{x}$ . (Or $\overline{x}$ - (c's -2.015) × 13.075/ $\sqrt{6}$			
	= -1.842(13).	A1	(e) $x^{-1}$ (c)			
	Refer to $t_5$ .	M1	No ft from here if wrong. P( $t < -1.842(13)$ ) = 0.0624.			
	Single-tailed 5% point is –2.015.	A1	P(t < -1.842(13)) = 0.0024. Must be minus 2.015 unless absolute values are being compared. No ft from here if wrong.			
	Not significant. Insufficient evidence to suggest that the amount of electricity used has decreased on average.	A1 A1	ft only c's test statistic. ft only c's test statistic. Conclusion in context to include "on average" o.e.	9		

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Q2Image: Constraint of the subgroups or strata that might (i)E1 E1 E1 E1 E1 E1 E1(i)There are identifiable subgroups or strata that might (i)E1 E1 E1 E1(ii)For each stratum× $\frac{2000}{79368}$ giving $79368$ giving $79368$ giving $813.9, 836.9, 245.4, 103.8$ so 814, 837, 245, 104M1 A1(ii)For each stratum× $\frac{2000}{79368}$ giving $813.9, 836.9, 245.4, 103.8$ so 814, 837, 245, 104M1 A1(iii)The population (or underlying distribution) is assumed to be symmetrical about its median.E2 E2, 1, 0. Award E1 for 2 out of 3 of the key features.(iii)H <sub>0</sub> : $m = 0$ H <sub>1</sub> : $m \neq 0$ where $m$ is the population median difference for the percentages.B1 B1B0th hypotheses in words only must include "population." For adequate verbal definition.(iii)H <sub>1</sub> : $m \neq 0$ where $m$ is the population median difference for the percentages.B1 B1 For differences. ZERO (out of 8) in this section if paired differences not used. For ranks. ft from here if ranks wrong. (or $W_+ = 1 + 3 + 4 + 6 + 9 + 10 = 33)$ W = 2 + 5 + 7 + 8 = 22M1 Result is not significant. No evidence to suggest a change in spending on average.M1 A1No ft from here if wrong. Result is not significant. No evidence to suggest a change in spending on average.A1 A1NoNo ft from here if wrong. A1 to only c's test statistic. H only c's	r	1										1
(i)exhibit different characteristics. Each stratum is randomly sampled. Use it o obtain a representative sample. Can get information on the individual strata.E1 E1 E1 E1(ii)For each stratum $\times \frac{2000}{79368}$ giving $813.9, 836.9, 245.4, 103.8$ so $814, 837, 245, 104$ M1 A1A1 All correct.2(b)The population (or underlying distribution) is assumed to be symmetrical about its median.E2 E2, 1, 0. Award E1 for 2 out of 3 of the key features.2(ii) $H_0: m = 0$ $H_1: m \neq 0$ where m is the population median difference for the percentages.B1 B1Both hypotheses. Hypotheses in words only must include "population". For adequate verbal definition.(iii) $H_0: m = 0$ $H_1: m \neq 0$ where m is the population median difference for the percentages.B1 B1Both hypotheses. Hypotheses in words only must include "population". For adequate verbal definition. $W = 2 + 5 + 7 + 8 = 22$ M1 Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 10$ . Lower (or upper if 33 used) 5% tail is 10 (or 45 if 33 used). Result is not significant. No evidence to suggest a change in spending on average.M1 A1No ft from here if wrong. i.e. a 2-tail test. No ft from here if wrong. A1 to only c's test statistic. A1	Q2											
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(i)assumed to be symmetrical about its median.key features.(ii) $H_0: m = 0$ $H_1: m \neq 0$ where m is the population median difference for the percentages.B1Both hypotheses. Hypotheses in words only must include "population". For adequate verbal definition. $\boxed{\text{Diff} -0.66 \ 0.02 \ -0.80 \ -0.91 \ 0.28 \ 0.76 \ 0.40 \ 1.68 \ -0.07 \ 1.12 \ Rank \ 5 \ 1 \ 7 \ 8 \ 3 \ 6 \ 4 \ 10 \ 2 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 1.12 \ 9 \ -0.000 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ 1.12 \ $		so 8	14, 8	37, 24	45, 1	04			A1	All correct.		2
$H_1: m \neq 0$ where $m$ is the population median difference for the percentages.B1only must include "population". For adequate verbal definition. $\boxed{\text{Diff} -0.66}$ $0.02$ $-0.91$ $0.28$ $0.76$ $0.40$ $1.68$ $-0.07$ $1.12$ $\boxed{\text{Rank} 5}$ $1$ $7$ $8$ $3$ $6$ $4$ $10$ $2$ $9$ $W_{-} = 2 + 5 + 7 + 8 = 22$ M1 B1For differences. ZERO (out of 8) in this section if paired differences not used. For ranks. A1 B1For ranks. ft from here if ranks wrong. (or $W_+ = 1 + 3 + 4 + 6 + 9 + 10 = 33$ )Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 10$ . Lower (or upper if 33 used) 5% tail is 10 (or 45 if 33 used). Result is not significant. No evidence to suggest a change in spending on average.M1 A1No ft from here if wrong. A1 A1A1 A1A1 ft only c's test statistic. ft only c's test statistic. ft only c's test statistic.10									E2		2 out of 3 of the	2
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Rank51783641029M1For differences. ZERO (out of 8) in this section if paired differences not used. $W = 2 + 5 + 7 + 8 = 22$ W = 2 + 5 + 7 + 8 = 22Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 10$ .Lower (or upper if 33 used) 5% tail is 10 (or 45 if 33 used).Result is not significant.No evidence to suggest a change in spending on average.M1KankM1M1M2M3M4M4M5M5M6M6M7M8M8M9M9M1M1M2M2M3M4M4M5M6M6M7M8M8M8M9M9M9M1M1M2M3M4M4M5M6M7M8M8M8M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9M9		Diff -0.66 0.02 -0.80 -0.91 0.28					0.76	0.40 1.68 -0.0	07 1.12			
$W = 2 + 5 + 7 + 8 = 22$ M1 A1 B1section if paired differences not used. For ranks. 			Rank		1	7	8	3	6	4 10 2	9	
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18	No evidence to suggest a cha					ge in spending on				ft only c's test statistic.		10
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				1
Q3				
(i)	Using mid- intervals 1.5, 1.7, etc	M1		
~ /				
	$\overline{x} = \frac{205}{100} = 2.05$	A1	Mean.	
	$s = \sqrt{\frac{425.16 - 100 \times 2.05^2}{99}} = 0.2227(01)$			
	s =	E1	s.d. Answer given; must show	3
	•		convincingly.	
(ii)	$f = 100 \times P(1.8 \le M < 2.0)$	M1	Probability $\times$ 100.	-
	$= 100 \times P(-1.1226 \le z < -0.2245)$			
	$=100 \times ((1 - 0.5888) - (1 - 0.8691))$	A1	Correct Normal probabilities. ft c's	
	$=100 \times (0.4112 - 0.1309) = 28.03$	A1	mean. Must show convincingly using Normal	3
			distribution. ft c's mean.	5
(iii)	H <sub>0</sub> : The Normal model fits the data.	B1	Ignore any reference to parameters.	
	$H_1$ : The Normal model does not fit the data.	B1		
		M1	Merge first 2 and last 2 cells.	
	$X^2 = 0.7294 + 0.1384 + 1.9623 + 3.5155 + 0.2437$	M1	Calculation of $X^2$ .	
	= 6.589(3)	A1	c.a.o.	
		241		
	Refer to $\chi_2^2$ .	M1	Allow correct df (= cells $- 3$ ) from wrongly grouped table and ft.	
			Otherwise, no ft if wrong.	
			$P(X^2 > 6.589) = 0.0371.$	
	Upper 5% point is 5.991.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Evidence suggests that the model does not fit the data.	A1	ft only c's test statistic. Conclusion in context.	9
	unu.		context.	
(iv)	The model			
	• overestimates in the $2.2 - 2.4$ class,	E1		
	<ul> <li>underestimates in the 2 – 2.2 class.</li> <li>At lower significance levels the test would not have</li> </ul>	E1 E1		3
	been significant.			5
	C C			
				18

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Q4				
(i)	-2 -1 1 2	G1 G1 G1	One (straight) line segment correct. Second (straight) line segment correct. Fully labelled intercepts + no spurious other lines.	3
(ii)	E(X) = 0 (By symmetry.)	B1		
	$E(X^{2}) = \int_{-1}^{0} x^{2} (1+x) dx + \int_{0}^{1} x^{2} (1-x) dx$ $= \left[\frac{x^{3}}{3} + \frac{x^{4}}{4}\right]_{-1}^{0} + \left[\frac{x^{3}}{3} - \frac{x^{4}}{4}\right]_{0}^{1}$	M1 M1	One correct integral with limits (which may be implied subsequently). Second integral correct (with limits) or allow use of symmetry.	
	$= 0 - \left(\frac{-1}{3} + \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - 0$ $= \frac{1}{6}$	M1	Correctly integrated and attempt to use limits.	
	:. $\operatorname{Var}(X) = \frac{1}{6} \left( -0^2 \right) = \frac{1}{6}$	A1	c.a.o. Condone absence of explicit evidence of use of $Var(X) = E(X^2) - E(X)^2$ .	5
(iii)	$\overline{L} \sim N\!\left(k, \frac{1}{300}\right)$	B1 B1 B1	Normal. Mean. Variance. ft c's variance in (ii) (> 0) / 50.	
	Normal distribution because of the Central Limit Theorem.	E1	Any reference to the CLT.	4
(iv)	CI is given by 90.06 ± 1.96 $\times \frac{1}{\sqrt{300}}$	M1 B1 M1		
	= 90.06 ± 0.11316= (89.947, 90.173)	A1	ft c's variance in (ii) (> 0) / 50. Must be expressed as an interval.	4
(v)	It is reasonable, because 90 lies within the interval found in (iv).	E1	Or equivalent.	1
				17